Further Pure 1 – June 2008

(1) $x^2 + x + 5 = 0$

a) $\alpha + \beta = -1$, $\alpha\beta = 5$

b) $\boxed{\text{SUM}}$ $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (-1)^2 - 2\times 5 = -9$

c) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2}{\alpha\beta} + \dfrac{\beta^2}{\alpha\beta}$

$= \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = \dfrac{-9}{5}$

d) $\boxed{\text{SUM}} = -9/5$

$\boxed{\text{PRODUCT}}$ $\dfrac{\alpha}{\beta} \times \dfrac{\beta}{\alpha} = \dfrac{\alpha\beta}{\alpha\beta} = 1$

$x^2 - \boxed{\text{SUM}}\,x + \boxed{\text{PRODUCT}} = 0$

$\Rightarrow x^2 + 9/5\,x + 1 = 0$

$\Rightarrow 5x^2 + 9x + 5 = 0$

(2) a) $z = x + iy$ $\quad z^* = x - iy$

$\Rightarrow 3i(x+iy) + 2(x - iy)$

$= 3ix - 3y + 2x - 2iy$

REAL : $2x - 3y$

IMAG : $3ix - 2iy$

b) REAL : $2x - 3y = 7$ (1)

IMAG : $3x - 2y = 8$ (2)

Simultaneous equations

$\boxed{(1)\times 3}$ $6x - 9y = 21$

$\boxed{(2)\times 2}$ $6x - 4y = 16$ $\quad -$

$-5y = 5 \Rightarrow y = -1$

Sub into (1) $2x - 3(-1) = 7$

$2x + 3 = 7$

$2x = 4 \Rightarrow x = 2$

$\Rightarrow z = 2 - i$ $\quad (z = x + iy)$

(3) a) $\int_9^\infty \frac{1}{\sqrt{x}}\,dx = \int_9^n x^{-1/2}\,dx$

$= \left[2x^{1/2}\right]_9^n = 2\sqrt{n} - 2\sqrt{9}$

$= 2\sqrt{n} - 6$

As $n \to \infty$, $2\sqrt{n} - 6$ does not approach a limit

$\therefore$ Integral has no value

b) $\int_9^\infty \frac{1}{x\sqrt{x}} = \int_9^n x^{-3/2}$

$= \left[-2x^{-1/2}\right]_9^n = \left[\frac{-2}{\sqrt{x}}\right]_9^n$

$= \frac{-2}{\sqrt{n}} + \frac{2}{\sqrt{9}} = \frac{-2}{\sqrt{n}} + \frac{2}{3}$

As $n \to \infty$, $\frac{-2}{\sqrt{n}} \to 0$

$\therefore$ Integral $\to 2/3$

(4) a) $y = ax + \frac{b}{x+2}$

$y(x+2) = a(x+2)x + b$

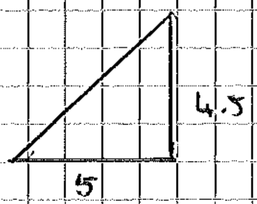
$\Rightarrow Y = aX + b$

b) i)

x	1	2	3	4
y	0.4	1.43	2.4	3.35
X	3	8	15	24
Y	1.2	5.72	12	20.01

ii) See Insert

iii) a = gradient



$\simeq \frac{4.5}{5}$

$\simeq 0.9$

b = intercept

≈ -2

(5) a) Radians: $\theta = 2n\pi \pm a$

Key angle $= \cos^{-1}(1/\sqrt{2}) = \pi/4 = a$

$\therefore \; x/2 + \pi/3 = 2n\pi \pm \pi/4$

$x/2 = 2n\pi - \pi/3 \pm \pi/4$

$\Rightarrow x = 4n\pi - 2\pi/3 \pm \pi/2$

b) Try values:

$\boxed{n=0}$ $\quad x = 0 - 2\pi/3 \pm \pi/2 = -\text{ve}!$ (both!)

$\boxed{n=1}$ $\quad x = 4\pi - 2\pi/3 - \pi/2 = 17\pi/6$

(6) a)

$$\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 0 & -4 \\ 4 & 0 \end{pmatrix}$$

b)

$$\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = 4I$$

c) $(AB)^2 =$

$$\begin{pmatrix} 0 & -4 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} 0 & -4 \\ 4 & 0 \end{pmatrix} = \begin{pmatrix} -16 & 0 \\ 0 & -16 \end{pmatrix} = -16I$$

$B^2 =$

$$\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$$

$$A^2B^2 = \begin{pmatrix}4&0\\0&4\end{pmatrix}$$

$$\begin{pmatrix}4&0\\0&4\end{pmatrix} \begin{pmatrix}16&0\\0&16\end{pmatrix} = 16I$$

$$\therefore (AB)^2 \neq A^2B^2$$

7 a) Left 1, up 7 $= \begin{pmatrix}-1\\7\end{pmatrix}$

b) i) $x = -1$

As $x \to \infty$, $y \to 7$ $\therefore$ $y = 7$

ii) when $\boxed{x = 0}$, $y = 7 + 1/1 = 8 \Rightarrow (0, 8)$

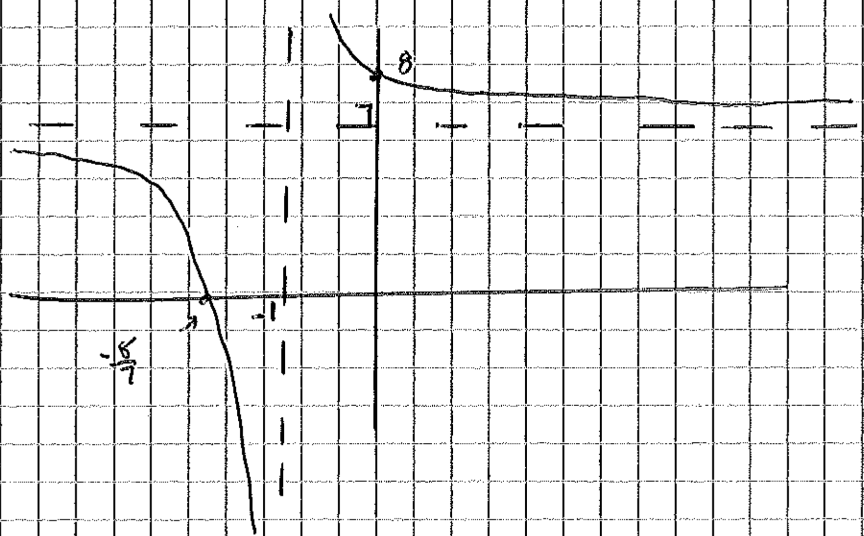
when $\boxed{y = 0}$ $\quad 0 = 7 + \frac{1}{x+1}$

$\Rightarrow 0 = 7(x+1) + 1$

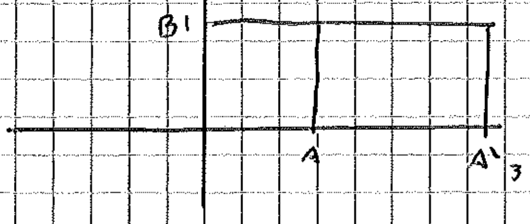
$\Rightarrow 0 = 7x + 7 + 1$

$\Rightarrow x = -8/7 \Rightarrow (-8/7, 0)$

c)



8 a) From Diagram: Stretch, SF 3, parallel to x-axis



$$= \begin{pmatrix}3&0\\0&1\end{pmatrix}$$

b) See Insert

c) $\boxed{1st}$ = stretch $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$

$\boxed{2nd}$ = Reflection in $y = x$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

order : $\boxed{2ND}$ $\boxed{1st}$ $\boxed{POINT}$

So, to find combined matrix:

$$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 3 & 0 \end{pmatrix}$$

(9) a) $x_1 = 3$

$y_1 = 4$

$m = m$

$$y - 4 = m(x - 3)$$

$$y - 4 = mx - 3m$$

b) Arrange eq of line to make $x =$

$$\rightarrow y - 4 = m(x - 3)$$

$$\frac{y-4}{m} = x - 3$$

$$\rightarrow x = \frac{y-4}{m} + 3$$

Sub into eq of parabola: $y^2 = 4\left[\frac{y-4}{m} + 3\right]$

$$y^2 = \frac{4y - 16}{m} + 12$$

$$my^2 = 4y - 16 + 12m$$

$$\rightarrow my^2 - 4y + 16 - 12m = 0$$

c) For tangents, $b^2 - 4ac = 0$

$$\rightarrow 16 - 4 \times m \times (16 - 12m) = 0$$

$$16 - 4m(16 - 12m) = 0$$

$16 - 64m + 48m^2 = 0$

$\div 16$ $\quad 1 - 4m + 3m^2 = 0$

$\rightarrow 3m^2 - 4m + 1 = 0$

$(3m - 1)(m - 1) = 0$

$m = 1/3 \qquad m = 1$

From Part a) $\quad y - 4 = mx - 3m$

$\boxed{m = 1/3} \rightarrow y - 4 = 1/3 x - 1$

$\rightarrow y = 1/3 x + 3$

$\boxed{m = 1} \rightarrow y - 4 = x - 3$

$\rightarrow y = x + 1$

d) Tangents touch parabolas at the 2 m values

$my^2 - 4y + 16 - 12m = 0$

$\boxed{m = 1/3}$ $\quad 1/3 y^2 - 4y + 12 = 0$

$\rightarrow y^2 - 12y + 36 = 0$

$(y - 6)(y - 6) = 0$

$y = 6 \qquad \boxed{y^2 = 4x}$

$x = 9$

$\rightarrow (9, 6)$

$\boxed{m = 1} \rightarrow y^2 - 4y + 4 = 0$

$(y - 2)(y - 2) = 0$

$y = 2 \qquad \boxed{y^2 = 4x}$

$x = 1$

$\rightarrow (1, 2)$